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HOLOGRAPHIC SPECTRAL ANALYSIS

Dr. T. L. Williamson
C. R. Lane

Electro-Optics Technology Branch
Electronic Technology Division

June 1979

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Charles Richard Lane

CHARLES RICHARD LANE, Project Engineer
Electro-Optics Techniques and Applications Group
Electro-Optics Technology Branch

Kenneth R. Hutchinson

KENNETH R. HUTCHINSON, Chief
Electro-Optics Techniques and Applications Group
Electro-Optics Technology Branch

FOR THE COMMANDER

Ronald F. Paulson

RONALD F. PAULSON, Acting Chief
Electro-Optics Technology Branch
Electronic Technology Division

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This report discusses the basic concepts and techniques required to design and use holographic optics for a number of spectral analysis problems. Since an in depth knowledge of the holographic processes is not required to use holographic optics for the various applications, the text is written for simplistic implementation into optical spectral analysis systems. Included in the report is a 9820A calculator ray trace program for aiding in the optical system design.

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TABLE OF CONTENTS

SECTION I

INTRODUCTION

Numerous applications arise where it is desirable to apply the spectral dispersive properties of holographic elements in order to perform a spectral decomposition. To design holograms for this type of application in most cases does not require an in-depth understanding of the holographic process. The following is a discussion of the basic concepts and techniques of sufficient detail to permit the reader to design and use holographic optics for a number of spectral analysis applications. Included is a calculator ray trace program for use on the Hewlett Packard 9820A calculator. This program can serve as a design aid for a large class of applications.

SECTION II

BASIC VECTOR RELATIONSHIPS

The field emanating from a point source can be represented by,

$$Ae^{i\vec{k}\cdot\vec{r}} \tag{1}$$

where A is the amplitude of the field, $\vec{k} = (2\pi/\lambda)\hat{k}$ is the propagation vector, and $\vec{r}$ is the vector from the point source to the point of observation. The phase relationship is described by the vector dot product

$$\vec{k} \cdot \vec{r} \tag{2}$$

In the construction of a hologram two such fields $\vec{k}_R$ and $\vec{k}_O$ (reference and object fields) exist simultaneously within the recording material. The resultant field is characterized by the vector relation

$$(\vec{k}_R - \vec{k}_O) \cdot \vec{r} = \vec{K} \cdot \vec{r} \tag{3}$$

where $\vec{K}$ is the vector describing the orientation of the resultant interference fringes.

The recording and processing of the imposed fringe pattern within the recording material results in a fringe pattern that is proportional to this incident interference pattern. (For a more complete description of this process in silver halide emulsions, see reference 1) i.e.,

$$\vec{K}_H = a(\vec{k}_R - \vec{k}_O)$$
$$= a\vec{K} \qquad (4)$$

where $\vec{K}_H$ is the vector describing the orientation of the recorded fringes and a is a constant reflecting the exposure and recording characteristics.

The process of reconstruction consists of illuminating the processed hologram (in the same angular orientation as recorded) with a wavefront that duplicates the reference wave. The resultant diffracted wavefront has a propagation vector given by,

$$\vec{k} = \vec{k}_R \pm n\vec{K} \qquad (5)$$

where n is the diffraction order. If the wavelength and geometry is identical to those of the construction process, the propagation vector will reduce to,

$$\vec{k} = n\vec{k}_O \text{ , } (1+n)\vec{k}_R - n\vec{k}_O \qquad (6)$$

Only the first term is of interest for this usage. The first diffracted order then results in a wavefront that is identical to the original object beam.

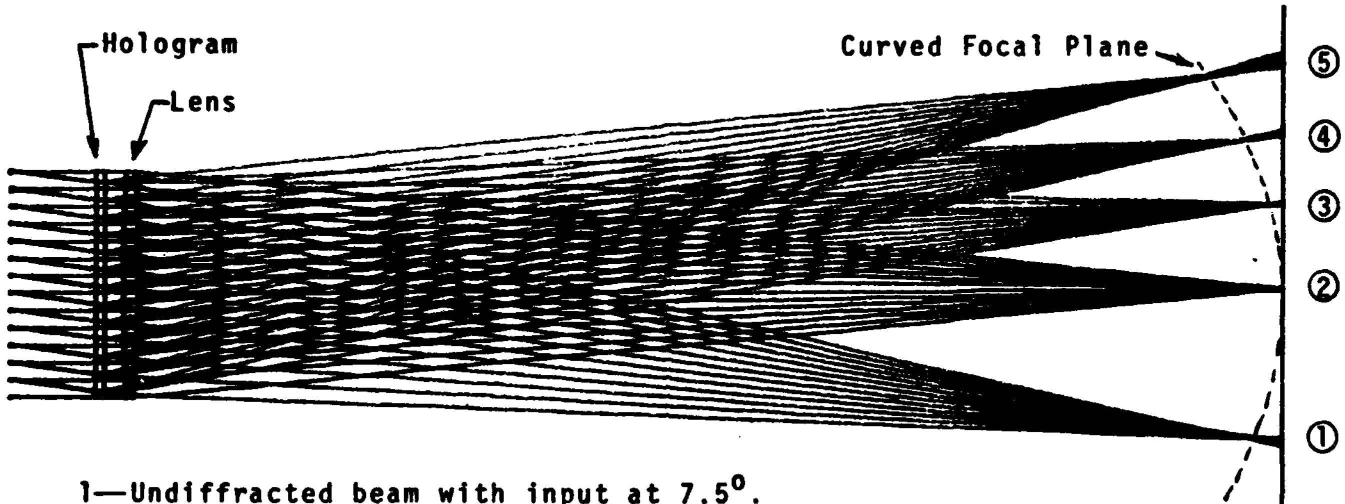
If a wavelength variation is included between the reconstruction and construction, the vector relationship becomes

$$\hat{k}(2\pi/\lambda_I) = (2\pi/\lambda_I)\hat{k}_R \pm n(2\pi/\lambda_E)\ \hat{K}$$
$$\hat{k} = \hat{k}_R \pm n(\lambda_I/\lambda_E)\ \hat{K}. \qquad (7)$$

where ^ indicates the unit vector and the subscripts E and I refer to exposure and illumination conditions. Thus for a given angular direction of the reconstructing field, the angle of the reconstructed ray is dependent on the illumination wavelength.

This analysis considers the fringe spacing to be a constant over the area of reconstruction. Through superposition, this can be extended to more complex holographic structures and is, in fact, quite general. The important result is that the change in propagation direction of the illumination that is induced by the hologram is proportional to the wavelength of the radiation. This result is illustrated in figure 1. This figure shows the change in focal position that results from various wavelengths of illumination when the holographic grating is placed in front of a simple lens. From this figure it is obvious that the plane containing the on axis focal points for the various wavelengths is curved. This will result in a design limitation if image resolution is to be maintained over a significant range of wavelengths.

One other feature that must be taken into account in the design is the fact that the multiple diffraction orders may overlap either due to spectral spread or spatial extent. The overall range of wavelengths is limited only by the angular range of the optics and the transmission characteristics of both the optics and the holographic materials. A considerable amount of flexibility exists due to the ability of varying the spatial frequency of the holographic fringes in the construction process.



1—Undiffracted beam with input at 7.5°.
2—Undiffracted beam with input at 0°.
3—Diffracted beam with input at 7.5° and wavelength of .4 microns.
4—Diffracted beam with input at 7.5° and wavelength of .5145 microns.
5—Diffracted beam with input at 7.5° and wavelength of .6328 microns.

FIGURE 1. Ray Trace of Simple Hologram- Lens System showing Axial Focus Positions for Several Illumination Wavelengths.

SECTION III

CONCLUSION

The proceeding discussion is meant to be only a first look at the use of holographic gratings as a tool to perform spectral decomposition. The discussion gives some of the basic vector relations that are used in the calculator ray trace program of the attached Appendix. This discussion together with the ray trace program will allow the reader to perform sufficient design analysis to answer the first questions associated with the use of holographic elements. For a more detailed treatment of the theory of holographic optics, the reader is referred to any of the current texts on the subject or the three dissertations listed as references.

APPENDIX

CALCULATOR RAY TRACE PROGRAM FOR SPHERICAL DIFFRACTIVE, REFRACTIVE AND REFLECTIVE OPTICAL ELEMENTS

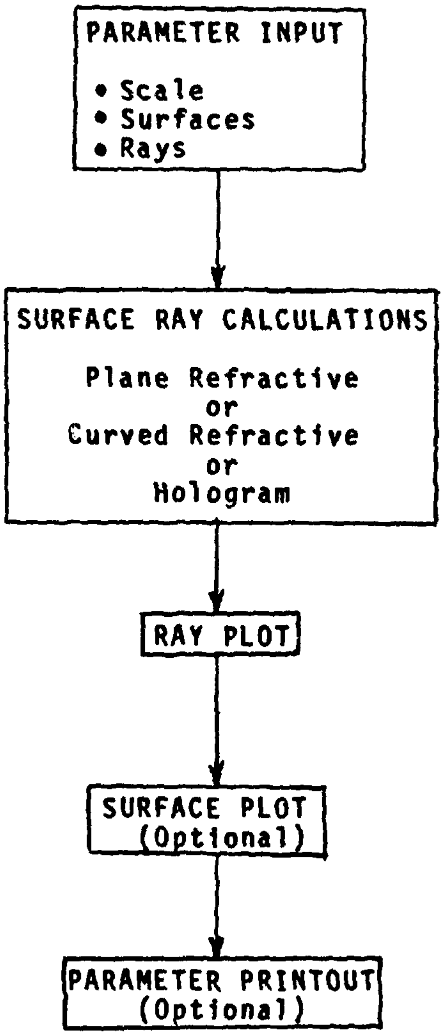
Introduction

Over the past several years programs have been written to simulate various aspects of hologram performance. These programs vary extensively in the degree of complexity depending on the flexibility and thoroughness required. Several programs for use with the table top (Hewlett-Packard 9820) calculator have existed in DHO alone. These programs are typically written for a particular problem and for this reason they are not general enough for widespread usage. This is an attempt to write and document a relatively simple program that is sufficient to be used for the analysis of a large number of optical problems. The program will handle refractive elements of spherical shape as well as reflective spherical elements and holographic elements. The program will provide a ray trace analysis of an optical system that is a combination of refractive or reflective elements together with one holographic lens.

The program uses Snell's law to calculate the ray direction across the refractive interfaces, and the vector relationship to determine the ray direction through diffractive elements. For reflective elements the index of refraction and the surface to surface distance must undergo a sign change at the reflective interface. This will be illustrated by an example in the ensuing discussion.

The following discussion gives a description of the program, its input requirements, the memory allocations, the calculations performed, examples of each type of element, and a printout of the program. The calculations are shown in mathematical form together with their equivalent statement in calculator form. This is provided for ease in trouble shooting as well as program modification. The program is written for use on the 9820A Hewlett-Packard calculator in conjunction with the 9862A Plotter, 9866A Printer, and the 9865A Cassette Memory.

PROGRAM FLOW DIAGRAM



INPUT PARAMETERS

SCALE: X_{min}, X_{max}, Y_{min}, Y_{max} .

NON-HOLOGRAM SURFACES:

R_D	Radius of curvature of the surface. (Positive if center of curvature is to the right of surface.)(0 = flat surf.)
TH	On axis distance to the next surface.
n	Index of refraction of the medium following the surface.
CA	Clear Aperture. (Radius of the active area of the surface.)

INPUT PARAMETERS
(continued)

HOLOGRAM PARAMETERS:

R/D Diverging wave focal distance from the hologram surface.

α/D Diverging wave angle with axis. (Positive if point source is above the axis)

R/C Converging wave focal distance from the hologram surface.

α/C Converging wave angle with axis. (Positive if point of convergence is below axis)

λ/I Illumination wavelength.

λ/E Exposure wavelength.

INPUT PARAMETERS
(continued)

INPUT RAY BUNDLE PARAMETERS:

CENTRAL-Y_1	Y position of the central ray in the input bundle.
DELTA-Y_1	Spacing between the parallel input rays.
A_{12}	Angle of rays between the first two surfaces. (Positive if the ray has a negative slope.)

MEMORY ALLOCATION

REGISTER	INFORMATION STORED
A	Surface number of hologram. (If none, A=0)
B	X_{min}
C	X_{max}
X	Y_{min}
Y	Y_{max}
Z	Index Register for Surfaces.
R(0)	Hologram R/D.
R(1)	Hologram α/D.
R(2)	Hologram R/C.
R(3)	Hologram α/C.
R(4)	Hologram LAMBDA/I.
R(5)	Hologram LAMBDA/E.
R(6)	
R(7)	Central Y_1.
R(8)	Delta Y_1 (From R(498)).
R(9)	Interim storage used in calc.

MEMORY ALLOCATION
continued

REGISTER	INFORMATION STORED
R(10Z)	Radius of curvature of surface Z.
R(10Z+1)	On axis distance to surface Z+1.
R(10Z+2)	Index of refraction between Z and Z+1.
R(10Z+3)	Radius of clear aperture of Z.
R(10Z+4)	Angle of incoming ray at plane Z.
R(10Z+5)	X position of Z plane on axis.
R(10Z+6)	X distance between intercepts of planes Z-1 and Z.
R(10Z+7)	X value at intercept point on plane Z.
R(10Z+8)	Y value at intercept point on plane Z.
R(10Z+9)	Angle of normal to surface at intercept point.
R(498)	Delta Y_1
R(507)	Interim calculations in Holo. sub.
R(508)	" " " " "
R(509)	" " " " "

DEFINITION OF TERMS

x_{Z0} — On axis x component of Z^{th} surface.

$d_{Z,Z+1}$ — X distance between the intercept on the Z and Z+1 surfaces.

$\alpha_{Z,Z+1}$ — Angle with respect to the x axis of the ray between surfaces Z and Z+1.

α_{RZ} — Angle of the normal to the Z^{th} surface at the point of intersection.

X_Z — x component of the intersection point on the Z^{th} surface.

Y_Z — y component of the intersection point on the Z^{th} surface.

n_Z — Index of refraction of the medium following the Z^{th} surface.

x'_{1Z} — x component of the center of curvature of the Z^{th} surface.

Calculations for a flat surface:

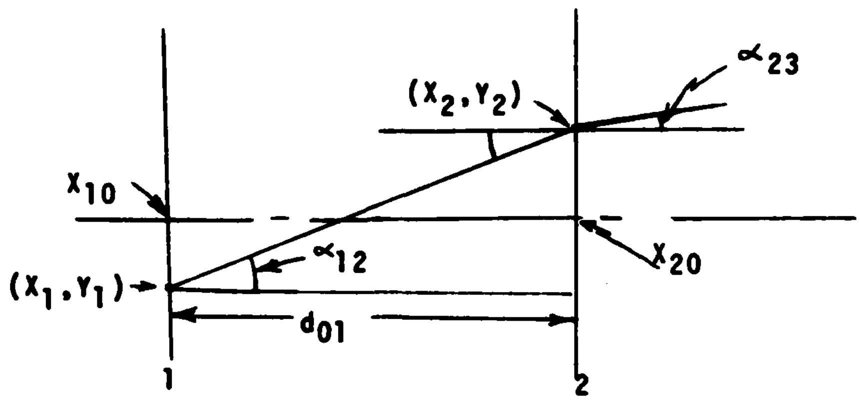


Figure A-1. Flat Surface Geometry

$$TH_1 + X_0 = X_{20}$$

R(Z+1) + R(Z+5) → R(Z+15)

$$X_{20} - X_1 = d_{01}$$

R(Z+15) - R(Z+7) → R(Z+16)

IF NEXT SURFACE IS FLAT
IF R(Z+10) = 0

$$Y_1 - d_{01} \tan\alpha_{12} = Y_2$$

R(Z+8) - R(Z+16)tanR(Z+4) → R(Z+18)

$X_1 + d_{01} = X_2$

R(Z+7) + R(Z+16) $\rightarrow$ R(Z+17)

IF NEXT SURFACE IS FLAT

IF R(Z+10) = 0

$\alpha_{R2} = 0$

0 $\rightarrow$ R(Z+19)

$\mathrm{Sin}^{-1}\ ((n_1/n_2)\mathrm{Sin}\,\alpha_{12}) = \alpha_{23}$

Sin^{-1}(R(Z+2)(SinR(Z+4))/R(Z+12)) $\rightarrow$ R(Z+14)

GO TO NEXT SURFACE

JMP 8

....

<u>Calculations for a spherical surface</u>:

The calculations for the ray intercept with a spherical surface require the simultaneous solution of the equation describing the ray path and that of

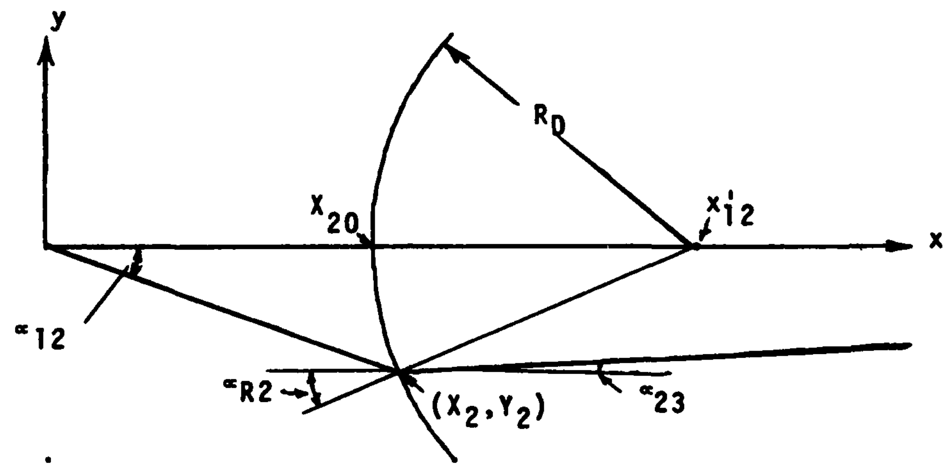


Figure A-2. **Geometrical Layout for the Spherical Surface Calculation.**

the spherical surface. The equation of the ray is,

$$y = mx + b$$

where

$$m = -\tan \alpha_{12}$$

and

$$b = Y_1 + X_1 \tan \alpha_{12}.$$

Then

$$y = -x \tan \alpha_{12} + (Y_1 + X_1 \tan \alpha_{12}).$$

The equation for a circular intercept whose center is located on the x axis is of the form;

$$(x - x'_{12})^2 + y^2 = R_{D2}{}^2$$

where

$$x'_{12} = x_{20} + R_{D2}$$

is the center. Thus

$$y^2 = R_{D2}{}^2 - x^2 + 2x'_{12}x - x'_{12}{}^2 .$$

Setting these equations for y equal and solving for the intercept point X_2 and letting

$$B = (x'_{12} + (Y_1 + X_1 \tan \alpha_{12}) \tan \alpha_{12}) \mathrm{Cos}^2 \alpha_{12}$$

and

$$C = (x'_{12} - R_{D2}^2 + (Y_1 + X_1 \tan \alpha_{12})^2) \mathrm{Cos}^2 \alpha_{12}$$

we have the familiar equation

$$x_2^2 - 2Bx_2 + C = 0$$

Thus

$$X_2 = B \pm (B^2 - C)^{1/2}$$

and

$$Y_2 = Y_1 + (X_1 - X_2)\tan \alpha_{12}.$$

The sign is determined by ;

$$\frac{R_{D2}}{|R_{D2}|} = \frac{R_{D2}}{(R_{D2}^2)^{1/2}} .$$

This solution is implemented in the program by the following steps.

$$X_{20} + R_{D2} + x'_{12}$$

$$R(Z+15) + R(Z+10) \rightarrow R9$$

Calculation of B

$$\cos^2 R(Z+4)(R9 + (R(Z+8)+R(Z+7)\tan R(Z+4))\tan R(Z+4)) \rightarrow R8$$

Calculation of C

$$\cos^2 R(Z+4)(R9^2 - R(Z+10)^2 + (R(Z+8)+R(Z+7)\tan R(Z+4))^2) \rightarrow R7$$

$$B - R_{D2}((B^2 - C)/R_{D2}{}^2)^{1/2} = X_2$$

$$R8 - R(Z+10)((R8^2 - R7)/R(Z+10)^2)^{1/2} \rightarrow R(Z+17)$$

$$Y_1 - (X_2 - X_1)\tan \alpha_{12} = Y_2$$

R(Z+8) - (R(Z+17)- R(Z+7))tanR(Z+4) $\rightarrow$ R(Z+18)

$$\sin^{-1}(Y_2/R_{D2}) = \alpha_{R2}$$

$\sin^{-1}$(R(Z+18)/R(Z+10)) $\rightarrow$ R(Z+19)

$$\alpha_{R2} + \sin^{-1}((n_1/n_2)(\sin(\alpha_{12} - \alpha_{R2}))) = \alpha_{23}$$

R(Z+19)+$\sin^{-1}$((R(Z+2)/R(Z+12))(Sin(R(Z+4)-R(Z+19))/R(Z+12)))

$\rightarrow$ R(Z+14)

GO TO NEXT SURFACE.

....

Calculations for a holographic element:

The calculations of the ray path through a holographic element are based on the following vector relations. The fringe pattern recorded in the hologram will have a localized fringe distribution whose orientation is described by the vector $\vec{K}$ where,

$$\vec{K} = \vec{k}_1 - \vec{k}_2 .$$

$\vec{k}_1$ and $\vec{k}_2$ are the wave propagation vector for the two fields used in the construction of the hologram.

The vector $\vec{K}$ can then be used to determine the direction of the diffracted ray upon reconstruction.

$$\vec{k}_R = \vec{k}_I - \vec{K}$$

where $\vec{k}_R$ and $\vec{k}_I$ refer to the reconstructed and illumination rays respectively. The program uses the direction sines for the wave vectors for the calculations.

The implementation is as follows;
Direction sine of the diverging field is

$$\mathrm{Sin}\theta_{12D} = (1/n_1)\mathrm{Sin}(\tan^{-1}((Y_1+R/D\mathrm{Sin}\alpha/D)/R/D\mathrm{Cos}\alpha/D))$$

$\text{Sin } \tan^{-1}((R(Z+8)+R0\text{Sin}R1)/R0\text{Cos}R1)/R(Z+2) \rightarrow R508$

The direction sine of the converging field is

$$\text{Sin}\theta_{12C} = (1/n_1)\text{Sin}\tan^{-1}((Y_1+R/C\text{Sin}\alpha/C)/R/C\text{Cos}\alpha/C).$$

$\text{Sin } \tan^{-1}((R(Z+8)+R2\text{Sin}R3)/R2\text{Cos}R3)/R(Z+2) \rightarrow R507$

Then, the diffracted ray angle is

$$\theta' = \text{Sin}^{-1}(\text{Sin}\alpha_{12} - (\lambda_I/\lambda_E)(\text{Sin}\theta_{12D} - \text{Sin}\theta_{12C}))$$

$$\text{Sin}^{-1}(\text{Sin}R(Z+4)-(R4/R5)(R508-R507)) \rightarrow R509$$

$$\alpha_{12} \rightarrow R510$$

$$R(Z+4) \rightarrow R510$$

$$(TH_1 + X_{10} = X_{20}),(X_{20} - X_1 = d_{01})$$

$$(R(Z+1) + R(Z+5) \rightarrow R(Z+15)) - R(Z+7) \rightarrow R(Z+16)$$

$$Y_1 - d_{01}\tan\theta' = Y_2$$

$$R(Z+8) - R(Z+16)\tan R509 \rightarrow R(Z+18)$$

$$X_1 + d_{01} = X_2$$

$$R(Z+7) + R(Z+16) \rightarrow R(Z+17)$$

IF $|n_1 \mathrm{Sin}\theta'/n_2| \leq 1$

IF ABS (R(Z+2)Sin R509/ R(Z+12) $\longrightarrow$ R(Z+14)) $\leq$ 1

$\mathrm{Sin}^{-1}(n_1 \mathrm{Sin}\theta'/ n_2) = \alpha_{23}$

Sin^{-1} R(Z+14) $\longrightarrow$ R(Z+14)

GO TO NEXT SURFACE.

....

Data Output:

The program is designed primarily to provide a ray trace plot as the output. This plot consists of a surface to surface plot of each of the rays in the prescribed ray bundle. An optional feature that exists is the plotting of the surfaces themselves. This subroutine is executed by the answer " 1 RUN PROGRAM" when the display "SURF.?" is shown. The plot of a given ray will be stopped if the intersection with the next surface exceeds the clear aperture range.

A second form of output is the tabular listing of the input surface parameters. This output is is included for record keeping and as a check of the accuracy of the input. The format of the data printout is shown in Figure A-3.

HOLO.AT SURF.NO.	R/D	ALPHA/D	R/C	ALPHA/C	LAMDA/I	LAMDA/E
..	...	......	...	....	....	....

SURF.NO.	RD	TH	N	CA
.	....	....	....	....

Figure A-3. Format for Data Printout.

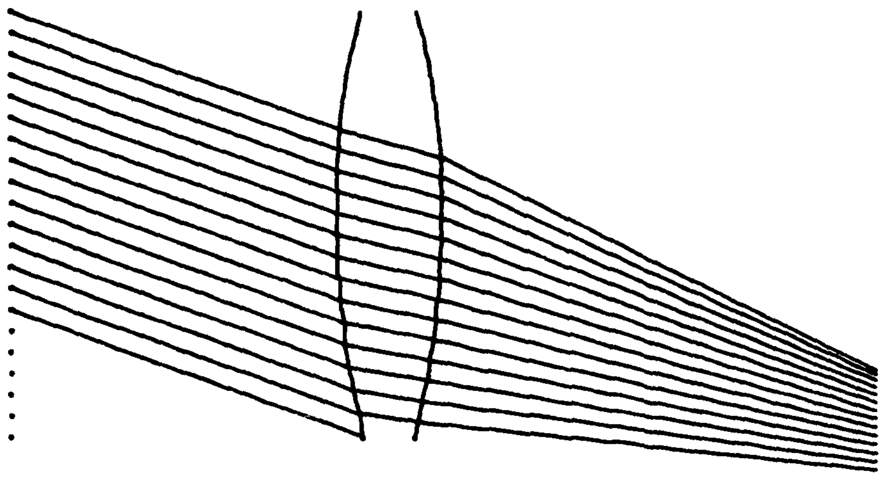


Figure A-4. Sample plot for a simple lens with an input ray bundle with spacing of .1 and an angle of 20^{o}. The surface parameters are shown in Figure A-5.

SURF.NO.	RD	TH	N	CA
1	0.000000	1.500000	1.000000	1.000000
2	4.000000	.500000	1.500000	1.000000
3	-4.000000	2.000000	1.000000	1.000000
4	0.000000	0.000000	1.000000	1.500000

Figure A-5. Data Printout for the Parameters Used in Figure A-4.

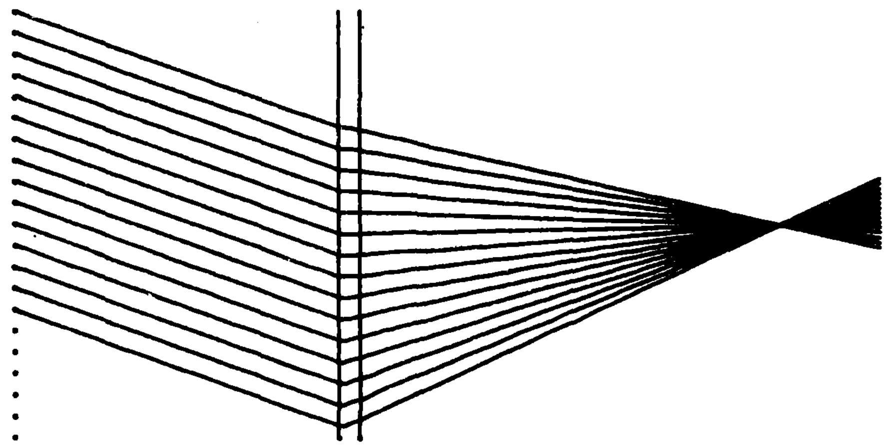


Figure A-6. **Sample Ray Trace through a Holographic Lens. Input Rays are the same as** Figure A-3.

HOLO.AT

SURF.NO.	R/D	ALPHA/D	R/C	ALPHA/C	LAMDA/I	LAMDA/E
2	1000.00	20.00	2.00	0.00	.6328	.6328

SURF.NO.	KD	TH	N	CA
1	0.000000	1.500000	1.000000	1.000000
2	0.000000	.100000	1.500000	1.000000
3	0.000000	2.400000	1.000000	1.000000
4	0.000000	0.000000	1.000000	1.500000

Figure A-7. Printout of the Parameters Used in Figure A-6.

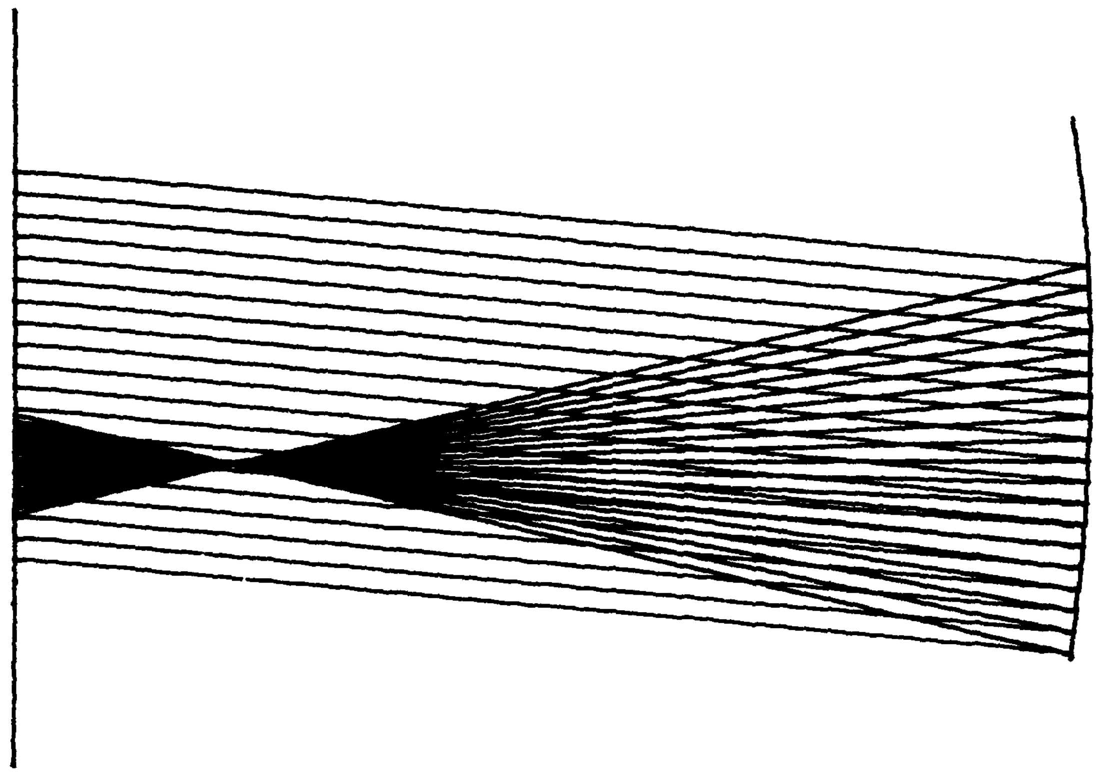


Figure A-8. Ray Trace for a Single Element Spherical Reflector.

SURF.NO.	RD	TH	N	CA
1	0.000000	5.000000	1.000000	1.000000
2	-8.000000	-5.000000	-1.000000	1.250000
3	0.000000	0.000000	1.000000	1.750000

Figure A-9. Input Parameters for the Ray Trace of Figure A-8.

```
0:
FXD 6;SFG 14;
ENT "SCALE,X MIN
",B,"X MAX",C,"Y
 MIN",X,"Y MAX",
Y⊢
1:
ENT "HOLO.AT SUR
F.NO?",A;IF A=0;
JMP 3⊢
2:
ENT "R/D=?",R0,"
ALPHA/D=?",R1,"R
/C=?",R2,"ALPHA/
C=?",R3⊢
3:
ENT "LAMDA/I=?",
R4,"LAMDA/E=?",R
5⊢
4:
10→Z⊢
5:
ENT "RD=?",RZ,"T
H=?",R(Z+1),"N=?
",R(Z+2),"CA=?",
R(Z+3)⊢
6:
IF R(Z+1)≠0;Z+10
→Z;JMP -1⊢
7:
ENT "CENTRAL Y1=
?",R7,"DELTA-Y1=
?",R498,"A12=?",
R14⊢
8:
R7-R13→R18;R498→
R8⊢
9:
0→R15→R16→R17→R1
9;10→Z⊢
10:
(R(Z+1)+R(Z+5)→R
(Z+15))-R(Z+7)→R
(Z+16);IF R(Z+10
)≠0;JMP 3⊢
11:
R(Z+8)-R(Z+16)
TAN R(Z+4)→R(Z+1
8);R(Z+7)+R(Z+16
)→R(Z+17)⊢
12:
0→R(Z+19);ASN (R
(Z+2)SIN R(Z+4)/
R(Z+12))→R(Z+14)
;JMP 8⊢
13:
R(Z+15)+R(Z+10)→
R(Z+19)⊢
14:
(R(Z+19)+(R(Z+8)
+R(Z+7)TAN R(Z+4
))TAN R(Z+4))
COS R(Z+4)↑2→R(Z
+18)⊢
15:
(R(Z+19)↑2-R(Z+1
0)↑2+(R(Z+8)+R(Z
+7)TAN R(Z+4))↑2
)COS R(Z+4)↑2→R(
Z+17)⊢
16:
R(Z+18)-R(Z+10)r
((R(Z+18)↑2-R(Z+
17))/R(Z+10)↑2)→
R(Z+17)⊢
17:
R(Z+8)-(R(Z+17)-
R(Z+7))TAN R(Z+4
)→R(Z+18)⊢
18:
ASN (R(Z+18)/R(Z
+10))→R(Z+19)⊢
19:
R(Z+19)+ASN (R(Z
+2)SIN (R(Z+4)-R
(Z+19))/R(Z+12))
→R(Z+14)⊢
20:
IF R(Z+1)≠0;Z+10
→Z;IF Z=10A;GSB
"HOLO"⊢
```

```
21:
IF R(Z+1)≠0;JMP
-11⊢
22:
SCL B,C,X,Y⊢
23:
10→Z⊢
24:
IF ABS R(Z+8)>R(
Z+3);JMP 3⊢
25:
PLT R(Z+7),R(Z+8
)⊢
26:
IF R(Z+1)≠0;Z+10
→Z;JMP -2⊢
27:
PEN ;R18+R8→R18;
IF R18≤R7+R13;
GTO 9⊢
28:
ENT "SURF.?",R10
00;IF R1000=1;
GSB "SURF."⊢
29:
ENT "PRINT OUT ?
",R1001;IF R1001
=1;GSB "PRINTER"
⊢
30:
GTO 7⊢
```

```
31:
"SURF."⊢
32:
20→Z⊢
33:
JMP 1;IF A≠0;IF
Z=10A+20;JMP 6⊢
34:
IF RZ=0;PLT R(Z+
5),-R(Z+3);PLT R
(Z+5),R(Z+3);
PEN ;IF R((Z+10→
Z)-9)≠0;JMP -1⊢
35:
-R(Z+3)→R15;IF R
(Z-9)=0;JMP 4⊢
36:
PLT R(Z+5)+RZ-RZ
√(1-R15↑2/RZ↑2),
R15⊢
37:
R15+R(Z+3)/20→R1
5;IF R15≤R(Z+3);
JMP -1⊢
38:
PEN ;Z+10→Z;IF R
(Z-9)≠0;JMP -5⊢
39:
RET ⊢
```

```
40:
"HOLO"⊦
41:
SIN ATN ((R(Z+8)
+R0SIN R1)/R0
COS R1)/R(Z+2)→R
508⊦
42:
SIN ATN ((R(Z+8)
+R2SIN R3)/R2
COS R3)/R(Z+2)→R
507⊦
43:
ASN (SIN R(Z+4)-
R4(R508-R507)/R5
)→R509;R(Z+4)→R5
10⊦
44:
(R(Z+1)+R(Z+5)→R
(Z+15))-R(Z+7)→R
(Z+16)⊦
45:
R(Z+8)-R(Z+16)
TAN R509→R(Z+18)
;R(Z+7)+R(Z+16)→
R(Z+17)⊦
46:
IF ABS (R(Z+2)
SIN R509/R(Z+12)
→R(Z+14))≤1;ASN
R(Z+14)→R(Z+14)⊦
47:
Z+10→Z;GTO 10⊦
48:
"PRINTER"⊦
49:
FMT 2/;WRT 8⊦
50:
IF A=0;JMP 7⊦
51:
FMT "HOLO.AT";
WRT 8⊦
52:
FMT "SURF.NO.",3
X,"R/D",3X,"ALPH
A/D",3X,"R/C",3X
,"ALPHA/C",3X,Z;
WRT 8⊦
53:
FMT "LAMDA/I",3X
,"LAMDA/E";WRT 8
⊦
54:
FMT FXD 4.0,FXD
11.2,FXD 8.2,2
FXD 8.2,2FXD 10.
4,FXD 11.4,FXD 1
0.2;WRT 8⊦
55:
WRT 8,A,R0,R1,R2
,R3,R4,R5⊦
56:
FMT 2/;WRT 8⊦
57:
FMT "SURF.NO.",9
X,"RD",16X,"TH",
16X,"N",17X,"CA"
,1/;WRT 8⊦
58:
10→Z⊦
59:
FMT FXD 4.0,4
FXD 18.6;WRT 8,Z
/10,RZ,R(Z+1),R(
Z+2),R(Z+3)⊦
60:
IF R(Z+1)≠0;Z+10
→Z;JMP -1⊦
61:
FMT 4/;WRT 8;
RET ⊦
62:
STP ⊦
63:
END ⊦
R1199
```

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